Stock Returns Under Alternative Volatility and Distributional Assumptions: The Case for India

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ABSTRACT

This paper has attempted studying the twin issues of asymmetry/leverage effect and excess kurtosis prevalent in India's stock returns under alternative volatility specifications as well as conditional distributional assumptions. This study has been carried out using daily-level data, based on India's premier stock index, BSESENSEX, covering India's post-liberalisation period from January 1996 to December 2010. Apart from lag returns, three other variables viz., call money rate, nominal exchange rate and daily dummies have been used as explanatory variables for specifying the conditional mean. Three alternative models of volatility representing the phenomenon of 'leverage effect' in returns viz., EGARCH, TGARCH and asymmetric PARCH along with standard GARCH have been considered for this study. As regards the assumption on conditional distribution for the innovations, apart from the Gaussian distribution, two alternative conditional distributions viz., standardized Student's distribution and standardized GED for capturing the leptokurtic property of the return distribution have been considered. Further, comparisons across these models have been done using forecast evaluation criteria suitable for both in-sample and out-of-sample forecasts. The results indicate that the asymmetric PARCH volatility specification performs the best in terms of both in-sample and out-of-sample forecasts. Also, the assumption of normality for the conditional distribution is not quite statistically tenable against the standardized GED and standardized Student's distribution for all the volatility models considered.

Key words: *Leverage Effect, Excess Kurtosis, Volatility Specification, Conditional Distribution, Out-Of-Sample Forecasts*
JEL Classifications: G00, G1, C5

1. INTRODUCTION AND LITERATURE REVIEW

Asset returns tend to cluster i.e., large (small) changes tend to follow large (small) changes. This phenomenon, called volatility clustering, was first observed by Mandelbrot (1963). Appropriate modelling of volatility is an essential part of any predictability study on asset returns. Moreover, proper estimation of volatility, for example, through the value-at-risk methodology, is necessary for portfolio analysis and risk management (Taylor, 2004).

As a consequence of volatility clustering, the hypothesis of normally distributed price changes, first noted by Bachelier (1900, see Herwartz, 2004, for details), failed to explain the

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unconditional distribution of empirical returns. This led to the generation of a vast body of literature on time-varying volatility, which began with the seminal contribution by Engle (1982). To capture the volatility of asset returns, Engle (1982) proposed the class of autoregressive conditional heteroscedastic (ARCH) models. Subsequently, the conclusions of most empirical studies indicate that in order to capture the dynamics of conditional variance, high orders of ARCH need to be selected. Therefore, to model a persistent movement in volatility without allowing for large number of coefficients in the ARCH model, Bollerslev (1986) suggested a generalization of this model called the generalized autoregressive conditional heteroscedastic (GARCH) model. The GARCH(1,1) model is the most commonly used model in the family of GARCH models. As pointed out by Bollerslev et al. (1994), the GARCH(1,1) model has indeed proven to be very useful in describing a wide variety of financial market data, including Indian Stock Market data (see Sarkar and Mukhopadhyay, 2005, for details). As evident from the specification of the GARCH model, this class of models is symmetric, in that negative and positive shocks have the same effect on volatility. Empirical literature on returns of risky assets, however, following Black (1976) and Christie (1982), has pointed out that negative shocks have a greater impact on future volatility than positive shocks. This asymmetry in equity returns is typically attributed to the 'leverage effect', which basically means that a fall in a firm's stock value causes the firm's debt to equity ratio to rise. This leads shareholders, who bear the residual risk of the firm, to perceive their future cash-flow stream as being relatively more risky.

Nelson (1991) extended the usual GARCH model in order to capture the leverage effect alongside volatility. This model, known as the exponential GARCH (EGARCH) model, has been widely applied in studies concerning stock market returns including a few on Indian stock returns. Building on the success of the EGARCH model, some other extensions of GARCH have been proposed to represent asymmetric responses in the conditional variance to positive and negative shocks. For instance, Glosten et al. (1993) and Zakoian (1994) have suggested using the threshold GARCH (TGARCH) model. Taylor (1986) and Schwert (1989) introduced the standard deviation GARCH model, where, instead of the variance, the standard deviation is modelled. This model, along with several other models, is generalized in Ding et al. (1993) with the 'power ARCH' specification. In the power ARCH (PARCH) model, the power parameter of the standard deviation can be estimated rather than imposed, and the optional parameters are added to capture asymmetry.

Apart from this issue of asymmetry in equity/stock returns, the issue of appropriate (conditional) distributional assumption of returns is also very important for analysing return data. Most often, conditional normality is considered to be the distribution, but unfortunately this is rarely supported by the volatility exhibited in economic and financial data. In fact, it is a widely accepted fact that most financial market data exhibit leptokurtosis and sometimes also asymmetry in return distributions (see, for example Kon, 1984; Mills, 1995 and Peiró, 1999).

As a solution to this problem within the framework of parametric models, alternative distributional assumptions allowing for excess kurtosis have been considered. For instance, Bollerslev (1987) and Tucker (1992), among others, have advocated evaluating sample log-likelihood under the assumption that innovations ε_t follow a standardized Student's *t*-distribution. In a similar spirit, Nelson (1991) suggested using a standardized general error distribution (GED).

The main objective in this paper is to consider the issues of leptokurtosis and asymmetry simultaneously for analysing the time series of returns on the well-known Indian stock market index called the Bombay stock exchange sensitive index (BSESENSEX). To the best of our knowledge, no such study considering the twin issues of "leverage effect" and excess kurtosis has so far been carried out with any Indian stock index or equity data. Only a few studies have so far been done on modelling asymmetric volatility alone: notable among these are the ones by Batra (2004) and Goudarzi and Ramanarayanan (2011). However, even these two are very limited in terms of the asymmetric volatility models considered and the methodology applied.

To that end, we have taken the three asymmetric volatility models mentioned earlier viz., EGARCH, TGARCH and asymmetric PARCH models along with the standard symmetric GARCH model, and three distributions of which one is the normal distribution and other two are the standardized *t*–distribution and standardized GED which capture excess kurtosis. This paper also compares these different volatility models and distributional assumptions in terms of forecast performance – both in–sample and out-of–sample -- by using appropriate forecast evaluation criteria.

Synthesizing these findings should reveal which volatility specification and distributional assumption fit the Indian stock returns best. This study examines daily-level BSESENSEX data covering India's post-liberalisation period from January 1996 to December 2010, of which observations from January 1996 to December 2008 constitute the in-sample period, and the remaining two years constitute the out-of-sample period.

This paper has been organized in the following format. The next section presents the modelling approach. Empirical results of the various models are discussed in Section 3. Section 4 presents the findings on forecast performances of these models. The paper closes with some concluding remarks in Section 5.

2. MODELLING APPROACH

In this section, we describe the modelling approach and the tests to be applied in our study. Assuming p_t to be the logarithm value of stock price index BSESENSEX, P_t, return r_t is defined as $r_t = p_t - p_{t-1}$. We first test the stationarity of r_t by applying the augmented Dickey-Fuller (ADF) test (Said and Dickey, 1984) and the Phillips-Perron test (1988). As the series is a long one, we test parameter stability using the Quandt-Andrews unknown breakpoint test, considering an autoregressive model of order 1, AR(1), for r_t. Along with lag returns and daily dummies, we have considered call money rate and daily nominal exchange rate of Indian rupee vis-à-vis US dollar as the other explanatory variables in the specification of the conditional mean (see, Sarkar and Mukhopadhyay, 2005, for details). We now specify the conditional mean model along with the four volatility models considered in this study.

2.1. Alternative Volatility Assumptions

Model for returns with GARCH(1,1) volatility: The specifications of the conditional mean and conditional variance of r_t are as follows:

$$r_t = \sum_{k=1}^{\tilde{m}} \varsigma_k r_{t-k} + \sum_{j=1}^{d} \beta_j D_{jt} + \omega i_t + \mu x_t + \varepsilon_t \qquad (2.1)$$

$$\varepsilon_t = z_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \sum_{j=1}^{d} \theta_j D_{jt} + \alpha_1 \varepsilon_{t-1}^{\ 2} + \delta h_{t-1} \tag{2.2}$$

Where z_t is the standardized innovation being independently and identically distributed (i.i.d.) with $E(z_t) = 0$ and $V(z_t) = 1$, h_t represents the conditional variance at time t, D_j, $j = 1,2,\ldots,d$, are daily dummies, $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\delta \geq 0$, i_t is the call money rate variable at time point t, x_t and is the log difference in nominal exchange rate.

Model for returns with TGARCH(1,1) volatility: The TGARCH(1,1) model is a simple extension of GARCH(1,1) model with an additional term added to take account of the possible leverage effect in the data. Obviously, the conditional mean specification is the same as in (2.1). The conditional variance is now specified as:

$$h_t = \alpha_0 + \sum_{j=1}^{d} \theta_j D_{jt} + \alpha_1 \varepsilon_{t-1}^{\ 2} + \delta h_{t-1} + \gamma \varepsilon_{t-1}^{\ 2} I_{t-1} \tag{2.3}$$

where I_{t-1} is an indicator function defined as: $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $= 0$ otherwise. For leverage effect in the returns, one would find $\gamma > 0$. The non-negativity restrictions on the other parameters are now $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\delta > 0$ and $\alpha_1 + \gamma \geq 0$, and For this model, positive news have an impact of α_1, while negative news have an impact of $(\alpha_1 + \gamma)$, and thus negative news have a greater impact on volatility if $\gamma > 0$.

Model for returns with EGARCH(1,1) volatility: In addition to TGARCH model, another well-known model designed to capture the asymmetries in the conditional variance is the EGARCH model introduced by Nelson (1991). In this model, h_t is expressed in the following logarithmic transformation:

$$\ln h_t = \alpha_0^{\ *} + \sum_{j=1}^{d} \theta_j D_{jt} + \delta^* \ln(h_{t-1}) + \gamma^* \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha_1^{\ *} \left(\left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - \sqrt{2/\pi} \right) \tag{2.4}$$

Now, if $\alpha_1^* > 0$, the process in (2.4) generates volatility clustering, provided that the quantity within the brackets is positive. In addition, in case γ^*, there will be a negative relationship between volatility and returns, which is the leverage effect. The model specified in (2.4) has an obvious advantage over the conditional GARCH model in that conditional variance in this specification will always be positive irrespective of the sign of the parameters since h_t is specified in logarithmic scale. However, as pointed out by Engle and Ng (1993), one particular drawback of the EGARCH model is that-- owing to the exponential structure of h_t -- the model may tend to overestimate the impact of outliers on volatility.

Model for returns with asymmetric PARCH(1,1) volatility: In the 'power ARCH' model, the power parameter λ of the conditional standard deviation σ_t ($=\sqrt{h_t}$) can be estimated rather than imposed, and the optional parameter ρ is added to capture asymmetry. The asymmetric PARCH(1,1) model is specified as follows:

$$\sigma_t^{\ \lambda} = \alpha_0 + \sum_{j=1}^{d} \theta_j D_{jt} + \delta' \sigma_{t-1}^{\lambda} + \alpha_1' (|\, \varepsilon_{t-1} \,| - \rho \varepsilon_{t-1})^{\lambda} \tag{2.5}$$

where $\lambda > 0$ and $|\rho| \leq 1$. The model is symmetric when $\rho = 0$. Note that if $\lambda = 2$ and $\rho = 0$ then the asymmetric PARCH model is simply the standard GARCH model.

2.2. Forecast Evaluation Criteria

Insofar as comparisons amongst the different models in terms of forecasts are concerned, we have used both the in-sample and out-of-sample forecast performances of returns, and the usual criteria have been applied, such as, the mean absolute error, mean absolute per cent error, the root mean squared error and the Theil's inequality (Theil). Further, in recent forecasting literature, some studies have been carried out to find an appropriate and accurate measure for evaluating different forecasting methods (see for example, Makridakis, 1993; Makridakis and Hibon, 1993; Makridakis et al., 2000). To that end, an adjusted form of mean absolute per cent error, which satisfies both theoretical and practical concerns while allowing for meaningful relative comparisons, has been proposed. The mean absolute error (MAE), which measures the average absolute forecast error, is defined as follows:

$$MAE = \frac{1}{n-n_1}\sum_{t=n_1+1}^{n} | r_{t+s} - f_{t,s} | \tag{2.6}$$

where $f_{t,s}$ is the s-step ahead forecast of returns based on t in-sample observations, n is the total sample size i.e., sum of in-sample and out-of-sample sizes, and $(n_1 + 1)$ *th* sample is the first out-of-sample forecast observation. The in-sample model estimation initially runs from observations 1 to n_1, and observations $n_1 + 1$ to n are available for out-of-sample forecasting so that the hold-out sample size is $n - n_1$.

The other two standard criteria i.e., the root mean squared error (RMSE) and the mean absolute per cent error (MAPE) are defined as:

$$RMSE = (\frac{1}{n-n_1}\sum_{t=n_1+1}^{n} (r_{t+s} - f_{t,s})^2)^{1/2} \tag{2.7}$$

$$MAPE = \frac{100}{n-n_1}\sum_{t=n_1+1}^{n} | \frac{r_{t+s} - f_{t,s}}{r_{t+s}} | \tag{2.8}$$

The RMSE is a conventional criterion that clearly weights greater forecast errors more heavily than smaller forecast errors in the forecast error penalty. This may, however, also be viewed as an advantage, if large errors are not disproportionately more serious, although the same critique could also be applied to the so-called least squares methodology. As these statistics in (2.6) and (2.7) are unbounded from above, little can be inferred based on the value of RMSE or MAE, when taken individually, as to which model would be considered best. Instead, the MAE or RMSE from one model should be compared with those of other models for the same data and forecast period, and the model with the lowest value of error measure would be considered as the better model. On the other hand, the MAPE criterion is scale invariant. As regards the Theil inequality coefficient, it always lies between 0 and 1, where zero indicates a perfect fit.

Finally, the adjusted MAPE (AMAPE) is defined as:

$$AMAPE = \frac{100}{n-n_1}\sum_{t=n_1+1}^{n} \frac{| r_{t+s} - f_{t,s} |}{(r_{t+s} + f_{t,s})/2}$$

This criterion corrects for the problem of asymmetry between the actual and forecast values.

2.3. Alternative Distributional Assumptions

For all the volatility specifications considered so far, namely, GARCH(1,1), EGARCH(1,1), TGARCH(1,1) and asymmetric PARCH(1,1), the parameters in the conditional mean as well as in the conditional variance are usually estimated by the maximum likelihood method under the assumption of conditional normality, which is more of an ad hoc assumption than one based on any statistical or economic reasoning. It has been found in the empirical literature on GARCH model that conditional normality of returns is more of an exception than the rule. Therefore, the issues regarding estimation and inference arising out of the distributional assumption of conditional non-normality of returns have also become prominent in the literature on volatility models. Further, it is worth noting that when the assumption of normality is violated, it is no longer possible to provide appropriate forecasting intervals. Moreover, maximum likelihood estimation, under misspecification of the (non-Gaussian) conditional distribution, may yield inconsistent parameter estimates (Newey and Steigerwald, 1997).

Under these circumstances, parametric models with other distributional assumptions have been suggested. Since excess kurtosis is an important problem with return data, distributions capturing excess kurtosis have been proposed. For instance, Bollerslev (1987) has advocated that innovations follow the standardized Student's *t*-distribution. On the other hand, Nelson (1991) has suggested the standardized general error distribution (GED). The probability density functions of these two alternative distributions are given below.

Standardized Student's *t*–distribution: The innovation z_t is said to follow a standardized Student's *t*–distribution with degrees of freedom v, mean zero, and variance h_t involving the parameters η if it has the following probability density function:

$$f(z_t, v) = \frac{\Gamma(\frac{v+1}{2})}{\Gamma(v/2)\sqrt{\pi(v-2)}}(1+\frac{z_t^{\ 2}}{(v-2)}) \tag{2.9}$$

where $\Gamma(.)$ denotes the gamma function, $\Gamma(s)=\int_0^\infty x^{s-1}\exp(-x)dx$, $s>0$. As $v\rightarrow\infty$, the density in (2.9) coincides with the Gaussian density.

Generalized error distribution (GED): The random variable ε_t, having mean zero and variance h_t involving the parameter vector η, is said to follow the generalised error distribution if its probability density function is given by:

$$f^*(\varepsilon_t \,|_{\eta,v}) = v\exp(-\frac{1}{2}\left|\frac{\varepsilon_t}{\lambda\sqrt{h_t}}\right|^v)[2^{\frac{v+1}{v}}\Gamma(\frac{1}{v})\lambda\sqrt{h_t}\,]^{-1} \tag{2.10}$$

where v is now called the shape parameter, and λ is defined as:

$$\lambda = [\frac{\Gamma(\frac{1}{v})}{2^{\frac{2}{v}}\Gamma(\frac{3}{v})}]^{1/2} \tag{2.11}$$

In case $v = 2$, the density in (2.10) reduces to that of $N(0,h_t)$, and the distribution becomes leptokurtic if $v < 2$. For $v = 1$, the GED coincides with the double exponential distribution. Further, as $v\rightarrow\infty$, the GED approximates the rectangular distribution.

3. EMPIRICAL RESULTS

The time series on India's premier stock index, BSESENSEX, has been collected from the official website of the Bombay Stock Exchange (www.bseindia.com). The data on call money rate have been taken from the website of the Reserve Bank of India (www.rbi.org.in), and the nominal exchange rate series have been obtained from the Federal Reserve Bank of St. Louis

of the U.S.A. (www.wikiposit.com). All data sets are at daily-level frequency from January 1996 to December 2010.
The different models, borne out of different assumptions on volatility and conditional distribution, considered in this paper have been estimated using the maximum likelihood method. All computations have been carried out with EVIEWS 7.

We present descriptive statistics in Table 3.1 for understanding the DGP of return series process. The mean and the standard deviation of the returns on BSESENSEX show that the mean is not different from zero. The value of the coefficient of skewness indicates that the series is skewed, and the kurtosis value (8.636) is much higher than that for a normal distribution (3), indicating that the return distribution is fat-tailed. We then present the results of the ADF and PP unit root tests on daily returns on BSESENSEX. Also included are the call money rate and the nominal exchange rate of Indian rupee in terms of the US dollar ($) - both at daily level.

Mean	0.001
Median	0.001
Std. Dev.	0.018
Skewness	-0.186
Kurtosis	8.636

Table 3.1 Descriptive statistics of daily returns on BSESENSEX.

Variable	ADF test statistic	*p*-value	PP test statistic	*p*-value
Return on BSESENSEX	-56.415	0.000	-56.398	0.000
Call money rate	-10.746	0.000	-27.593	0.000
Return on Exchange rate	-63.948	0.000	-63.836	0.000

Table 3.2 Results of unit root tests.
Notes: The time series on call money rate is at level value. The SIC criterion has been used to choose the lag value for the ADF estimating equation. The MacKinnon (1996) one-sided *p*-values are reported.

Test	Statistic value	*p*-value
Maximum LR *F*-statistic	3.157	0.865
Maximum Wald *F*-statistic	6.314	0.372
Exp LR *F*-statistic	0.598	0.787
Exp Wald *F*-statistic	1.255	0.402
Ave LR *F*-statistic	1.139	0.716
Ave Wald *F*-statistic	2.279	0.311

Table 3.3 Results of Andrews test for structural break in the return series.
Notes: *p*-values have been calculated using Hansen's (1997) method, and the truncation parameter has been taken to be 0.15.

The values of these test statistics are presented in Table 3.2. As evident from this table, all the series are stationary at the 1% level of significance. Also, the kurtosis value of BSESENSEX returns is 8.636, indicating fat tails compared to that of normal distribution. The significant value of the Jarque-Bera test statistic clearly rejects the assumption of normality.

Furthermore, the values of Ljung-Box test statistic for the squared returns are significant, which indicates the presence of second-order dependence in returns. For instance, the values for lags 2, 12 and 22 are 195.57, 703.69 and 920.32, respectively, which are all highly significant. An application of the Andrews (1993) test of parameter stability reveals that the return series is stable for the entire period considered. The results of this test for stability are presented in Table 3.3. As evident from Table 3.3, statistic values derived from the Andrews test suggest that the hypothesis of 'no structural break' cannot be rejected.

We also conducted a one-sample Kolmogorov-Smirnov test for normality and found that the return series shows departure from normality, as shown below in Table 3.4.

Smaller group	Test Statistic	*p*-value	Corrected *p*-value
Return on BSESENSEX	0.056	0.000	-
Cumulative	-0.059	0.000	-
Combined K-S	0.059	0.000	0.000

Table 3.4 One-sample Kolmogorov-Smirnov test against theoretical distribution normal.

Figure 3.1 Histogram of BSESENSEX returns series

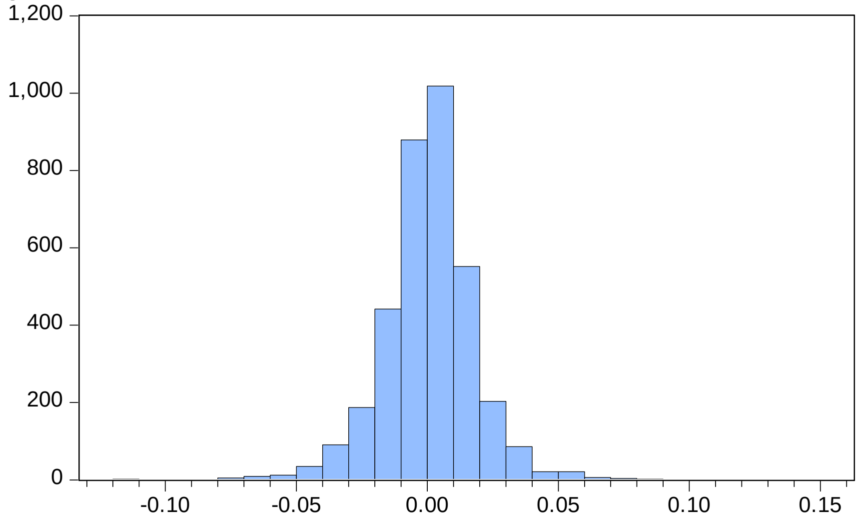


Figure 3.2 P.d.f. of standard normal and standardized Student's *t*–distribution of BSESENSEX series

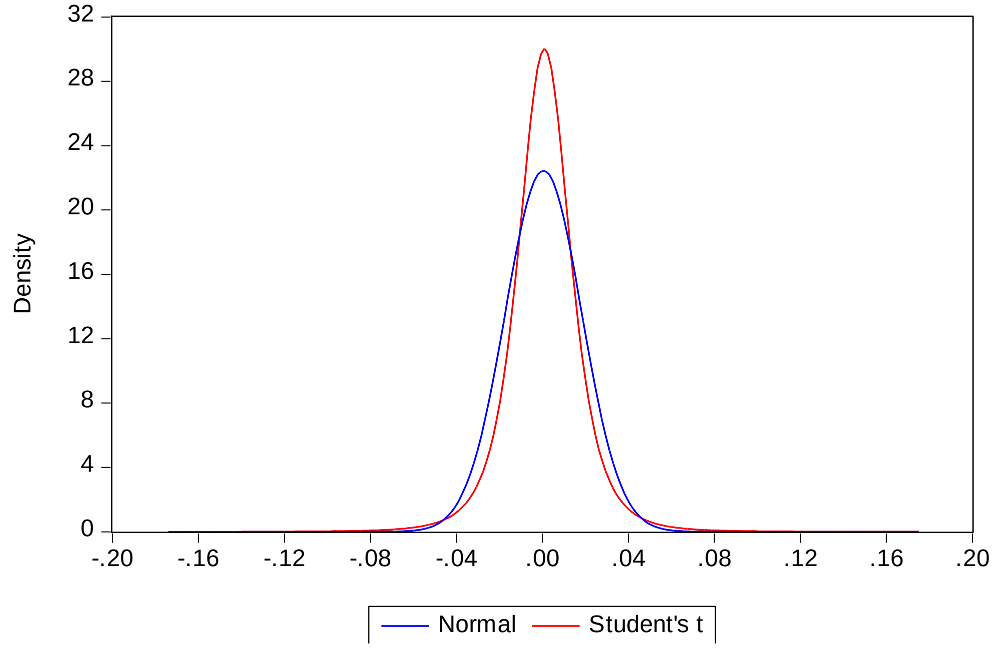
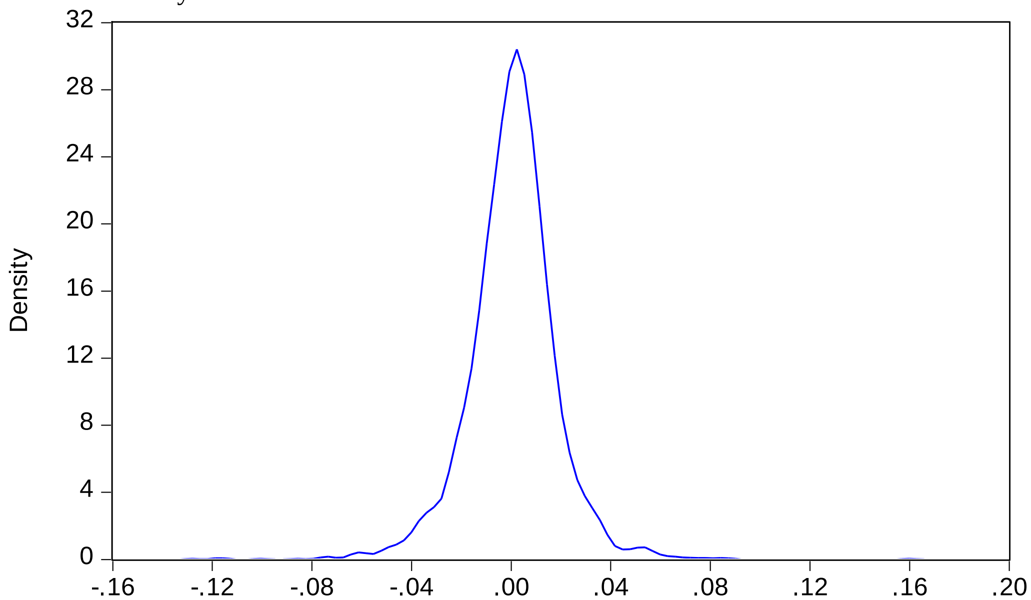


Figure 3.3 Kernel density of BSESENSEX returns



The histogram of the return series is given in Figure 3.1 demonstrating the asymmetry of the series. The Kernel density estimate, as shown in Figure 3.3, also confirms departure from normality, indicating that the excess kurtosis is strongly evident for this series. Similar conclusions can also be drawn from Figure 3.2.

Next, we report the results of the maximum likelihood (ML) method of estimation of the conditional mean as specified in equation (2.1) and the conditional variance given by the GARCH family of volatility specification (i.e., equation 2.2 for GARCH(1,1)) under the three assumed conditional distributions viz., Gaussian, *t* and GED in Table 3.5. The order of the GARCH volatility model is (1,1) for all distributional assumptions. As regards the choice of $\tilde{m}$, own lag length of r_t, we began with a sufficiently large value of 30. Thereafter, insignificant lag values of r_t were dropped, and the model was re-estimated with only the significant lags. The results of this model are reported in Table 3.5. This has been followed for all the models.

Apart from these lags, the interest rate (i_t) and nominal exchange rate (x_t), both of negative signs, were found to be significantly present. This strong evidence of the causal influence of exchange rate on stock prices is consistent with similar findings (see, for instance, Abdalla and Murinde, 1997 and Granger et al., 2000). The main implication is that changes in exchange rate affect a firm's exports as well as the cost of imported goods and production inputs, thus ultimately affecting stock prices. Since interest rate is an opportunity cost of holding stock, an increase in the interest rate is likely to lead to a substitution effect between stocks and other interest bearing assets. Therefore, as interest rate declines, stock price could be expected to rise (see Musilek, 1997, for further details).

Moreover, an LM test (also called Rao's score test) was carried out to test for the presence of any remaining volatility in the residuals of the fitted model under each of the three different assumptions on the conditional distribution. This test statistic is evaluated under the null hypothesis of 'no conditional heteroscedasticity' in the fitted residuals $\tilde{\varepsilon}_t$. In case the alternative hypothesis is an ARCH(q) model, $\tilde{\varepsilon}_t^{\,2}$ is regressed on $\tilde{\varepsilon}^2_{t-1},\ldots,\tilde{\varepsilon}^2_{t-q}$, and the test statistic is obtained as nR^2, where R^2 is the coefficient of multiple determination of the above regression. The values of this test statistic have been obtained as 1.883, 2.158 and 0.368, with their corresponding p-values being 0.170, 0,142 and 0.543 under normality, t and GED distributions, respectively. Thus, it may be concluded that GARCH(1,1) is adequate for the return series since there is no volatility in its residuals.

Variable	$N(0,h_t)$	$t(v,0,h_t)$	GED$(0,h_t)$
	Conditional mean		
r_{t-1}	0.081 (4.162)*	0.080 (4.343)*	0.076 (4.153)*
r_{t-3}	0.032 (1.761)***	–	–
r_{t-5}	-0.052 (-2.803)*	-0.050 (-2.816)*	-0.052 (-2.927)*
r_{t-6}	-0.055 (-3.119)*	-0.049 (-2.829)*	-0.049 (-2.870)*
r_{t-9}	0.034 (1.967)**	–	–
r_{t-11}	–	-0.033 (-2.016)**	-0.031 (-1.913)***
r_{t-18}	-0.029 (-1.797)***	–	-0.029 (-1.719)***
r_{t-19}	-0.043 (-2.714)*	-0.028 (-1.776)***	-0.036 (-2.329)**
i_t	-0.01×10^{-2} (2.373)**	-0.02×10^{-2} (-3.001)*	-0.02×10^{-2} (-3.025)*
D_1	0.003 (4.212)*	0.003 (4.433)*	0.003 (4.867)*
D_2	0.002 (3.086)*	0.003 (4.173)*	0.003 (4.157)*
D_3	0.002 (3.318)*	0.002 (3.479)*	0.002 (3.667)*
D_4	0.003 (3.779)*	0.003 (4.361)*	0.003 (4.463)*
D_5	0.002 (2.783)*	0.002 (2.637)*	0.002 (2.850)*
X_t	-0.716 (-10.293)*	-0.707 (-10.031)*	-0.729 (-10.232)*
	Conditional variance		

α_0	0.012×10^{-3} (2.437)**	0.002×10^{-2} (5.655)*	0.002×10^{-2} (6.830)*
$\varepsilon_{t-1}^{\ 2}$	0.158 (17.181)*	0.144 (8.725)*	0.146 (10.192)*
h_{t-1}	0.807 (74.504)*	0.827 (47.184)*	0.819 (51.726)*
D_1	0.003×10^{-2} (2.819)*	–	–
D_2	-0.005×10^{-2} (-4.898)*	-0.006×10^{-2} (-3.922)*	-0.006×10^{-2} (-4.566)*
D_3	0.002×10^{-2} (2.055)**	–	–
$\hat{v}$ (Distribution parameter)	–	7.219 (9.396)*	1.423 (36.801)*
Max log likelihood	8490.319	8558.595	8543.195

Table 3.5 Estimates of parameters in conditional mean and conditional variance of GARCH(1,1) volatility model under alternative distributional assumptions.
Notes: Coefficients of variables found to be significant in at least one model have been reported. Blank entry means that the corresponding variable has been found to be insignificant under the concerned distribution. The values in parentheses indicate the corresponding *t*-statistic values. *, ** and.*** indicate significance at (i) 1%, (ii) 5% and (iii) 10% levels of significance , respectively.

Variable	$N(0,h_t)$	$t(v,0,h_t)$	$GED(0,h_t)$
	Conditional mean		
r_{t-1}	0.087 (4.521)*	0.086 (4.580)*	0.081 (4.364)*
r_{t-5}	-0.050 (-2.720)*	-0.044 (-2.522)**	-0.045 (-2.589)*
r_{t-6}	-0.049 (-2.876)*	-0.044 (-2.607)*	-0.045 (-2.674)*
r_{t-9}	0.034 (1.990)*	–	–
r_{t-19}	-0.043 (-2.772)*	-0.030 (-1.926)***	-0.039 (-2.542)*
i_t	-0.01×10^{-2} (-2.071)**	-0.02×10^{-2} (-2.794)*	-0.02×10^{-2} (-2.866)*
D_1	0.002 (3.485)**	0.003 (3.944)*	0.003 (4.334)*
D_2	0.001 (2.052)**	0.002 (3.326)*	0.002 (3.303)*
D_3	0.002 (2.488)*	0.002 (2.893)*	0.002 (3.023)*
D_4	0.002 (2.634)*	0.002 (.5186)*	0.002 (3.584)*
D_5	0.001 (1.934)***	0.001 (2.115)**	0.002 (2.303)**
X_t	-0.685 (-10.035)*	-0.677 (-9.673)*	-0.692 (-9.838)*
	Conditional variance		
α_0	0.002×10^{-2} (3.136)*	0.003×10^{-2} (6.319)*	0.003×10^{-2} (7.463)*
$\varepsilon_{t-1}^{\ 2}$	0.067 (6.278)*	0.070 (4.153)*	0.069 (4.441)*
$\varepsilon_{t-1}^{\ 2} I_{t-1}$	0.185 (10.288)*	0.160 (5.782)*	0.167 (6.487)*
h_{t-1}	0.784 (63.696)*	0.802 (42.077)*	0.793 (45.184)*
D_1	0.003×10^{-2} (3.046)*	–	–
D_2	-0.005×10^{-2}	-0.005×10^{-2}	-0.006×10^{-2}

	(-4.317)*	(-3.751)*	(-4.277)*
D_3	0.002×10^{-2} (1.913)***	–	–
$\hat{\nu}$ (Distribution parameter)	–	7.751 (8.674)*	1.466 (34.834)*
Max log likelihood	8518.586	8574.205	8559.084

Table 3.6 Estimates of parameters in conditional mean and conditional variance of TGARCH(1,1) volatility model under alternative distributional assumptions.

Notes: Coefficients of variables found to be significant in at least one model have been reported. Blank entry means that the corresponding variable has been found to be insignificant under the concerned distribution. The values in parentheses indicate the corresponding *t*-statistic values. *, ** and *** indicate significance at (i) 1%, (ii) 5% and (iii) 10% levels of significance, respectively.

We now discuss the presence of asymmetry in volatility of Indian stock returns. To that end, we first look at the computational figures presented in Table 3.6. Thus, in case of TGARCH model for volatility with Gaussian conditional distribution, the ML estimate of the term capturing leverage effect i.e., $\varepsilon_{t-1}^2 I_{t-1}$ is found to be 0.185, and its t–statistic value is 10.288, which is highly significant at 1% level. This term, representing asymmetry, is highly significant and positive irrespective of the distributional assumptions considered. Thereby, we can conclude that volatility increases more after a large negative shock than after an equally

Variable	$N(0,h_t)$	$t(v,0,h_t)$	$GED(0,h_t)$
Conditional mean			
r_{t-1}	0.081 (4.404)*	0.078 (4.254)*	0.074 (4.070)*
r_{t-5}	-0.049 (-2.838)*	-0.046 (-2.678)*	-0.048 (-2.790)*
r_{t-6}	-0.048 (-2.828)*	-0.042 (-2.534)*	-0.043 (-2.587)*
r_{t-9}	0.039 (2.366)*	–	–
r_{t-19}	-0.042 (-2.882)*	-0.028 (-1.820)***	-0.038 (-2.576)*
i_t	–	-0.02×10^{-2} (-2.747)*	-0.02×10^{-2} (-2.743)*
D_1	0.002 (3.367)*	0.003 (4.073)*	0.003 (4.425)*
D_2	–	0.002 (3.141)*	0.002 (3.046)*
D_3	0.001 (1.668)***	0.002 (2.886)*	0.002 (3.017)*
D_4	0.001 (1.857) ***	0.002 (3.500)*	0.002 (3.511)*
D_5	–	0.001 (1.812)***	0.001 (1.847)***
X_t	-0.702 (-10.531)*	-0.696 (-10. 150)*	-0.708 (-10.266)*
Conditional variance			
α_0^*	-0.855 (-11.326)*	-0.649 (-7.020)*	-0.724 (-7.479)*
$\left\lvert \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right\rvert$	0.296 (16.635)*	0.281 (10.403)*	0.286 (11.270)*
$\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$	-0.112 (-12.241)*	-0.099 (-6.313)*	-0.104 (-7.198)*
$\ln(h_{t-1})$	0.922 (118.811)*	0.941 (97.010)*	0.933 (91.281)*

D_1	0.132 (2.355)**	–	–
D_2	–	-0.292 (-3.397)*	-0.293 (-3.676)*
D_3	-0.231 (-3.913)*	–	–
$\hat{\nu}$ (Distribution parameter)	–	7.636 (8.585)*	1.464 (33.740)*
Max log likelihood	8514.950	8572.594	8557.529

Table 3.7 Estimates of parameters in conditional mean and conditional variance of EGARCH(1,1) volatility model under alternative distributional assumptions.
Notes: Coefficients of variables found to be significant in at least one model have been reported. Blank entry means that the corresponding variable has been found to be insignificant under the concerned distribution. The values in parentheses indicate the corresponding *t*-statistic values. *, ** and *** indicate significance at (i) 1%, (ii) 5% and (iii) 10% levels of significance, respectively.

large positive shock in the Indian stock market. The *t*–statistic values of the coefficient corresponding to this term for *t*–distribution and GED are 5.782 and 6.487, respectively, and as indicated in Table 3.6, both are highly significant. We note from Table 3.6 that most of the variables including lag values of return in the conditional mean, which are significant with TGARCH(1,1) specification, are similar to those attained with the GARCH(1,1) volatility model. Furthermore, their respective signs are the same in the two models. We also find that the maximized log-likelihood value with TGARCH model is always higher compared to the corresponding GARCH model, and this is true for all the distributional assumptions. For instance, the maximized log-likelihood value for TGARCH(1,1) model with normal distribution is 8518.586 as compared to 8490.319 for GARCH(1,1) with normal distribution.

Variable	$N(0,h_t)$	$t(v,0,h_t)$	$GED(0,h_t)$
	Conditional mean		
r_{t-1}	0.096 (5.244)*	0.092 (4.964)*	0.084 (4.581)*
r_{t-3}	0.041 (2.332)**	0.0362 (2.015)**	0.032 (1.826)***
r_{t-4}	0.039 (2.214)**	0.038 (2.113)**	0.039 (2.258)**
r_{t-5}	-0.045 (-2.537)**	-0.043 (-2.430)**	-0.047 (-2.695)*
r_{t-6}	-0.044 (-2.624)*	-0.042 (-2.524)**	-0.045 (-2.676)*
r_{t-8}	–	0.033 (1.943)***	0.038 (1.998)**
r_{t-19}	-0.036 (-2.412)**	-0.027 (-1.784)***	-0.037 (-2.422)**
i_t	–	-0.008×10^{-2} (-1.677)***	-0.01×10^{-2} (-2.394)**
D_1	0.002 (3.277)*	0.002 (3.218)*	0.003 (3.961)*
D_2	–	0.001 (1.823)***	0.002 (2.300)**
D_3	–	0.001 (1.857)***	0.002 (2.391)**
D_4	–	0.002 (2.461)**	0.002 (2.913)**
D_5	–	–	0.001 (1.696)***
X_t	-0.684	-0.677	-0.694

	(-9.818)*	(-9.599)*	(-9.763)*
Conditional Variance			
α_0	0.03×10^{-2}	0.01×10^{-2}	0.01×10^{-2}
	(1.530)	(1.052)	(1.044)
α_1	0.157	0.147	0.150
	(13.681)*	(8.227)*	((8.808)*
ρ	0.460	0.406	0.398
	(12.235)*	(5.929)*	(6.676)*
δ	0.790	0.817	0.805
	(60.263)*	(44.167)*	(44.062)*
λ	1.391	1.462	1.476
	(9.167)*	(6.599)*	(6.617)*
$\hat{\nu}$ (Distribution parameter)	_	7.669	1.455
		(8.764)*	(34.404)*
Max log likelihood	8508.688	8574.038	8559.898

Table 3.8 Estimates of parameters in conditional mean and conditional dispersion of PARCH(1,1) volatility model under alternative distributional assumptions.

Notes: Coefficients of variables found to be significant in at least one model have been reported. Blank entry means that the corresponding variable has been found to be insignificant under the concerned distribution. The values in parentheses indicate the corresponding *t*-statistic values. *, ** and *** indicate significance at (i) 1%, (ii) 5% and (iii) 10% levels of significance , respectively.

We now check whether the estimation results from the EGARCH(1,1) specification, presented in Table 3.7, also indicate the presence of asymmetry in the volatility of Indian stock returns. We note from the entries of this table that the results are similar to those obtained from the GARCH specification. In the return series, the asymmetric term γ^* is negative and significant for all the distributional assumptions. For example, for returns with normal distribution, the ML estimate of γ^* is –0.119, and its *t*–statistic value is 12.241, which clearly indicates significance at 1% level of significance. As ‘leverage effect’ or asymmetry in volatility is found to be significantly present in both the TGARCH and EGARCH models for all three distributions, we can infer that asymmetric response of volatility to positive and negative shocks is an important phenomenon for the Indian stock market.

The findings for asymmetric PARCH volatility model reported in Table 3.8, also show that the estimate under normal conditional distribution of the ρ parameter denoting asymmetry is 0.460, which is significant. This term representing asymmetry is significantly present for other distributions as well. One can further note that the maximum log-likelihood values for TGARCH, EGARCH and PARCH models are almost the same for each of the three distributions and always higher than that of the GARCH model, although the value is somewhat smaller for EGARCH model. This discrepancy in the maximum log-likelihood values suggests that the TGARCH and asymmetric PARCH models are better representations for volatility in returns on BSESENSEX, compared to the EGARCH model.

Focusing now on the issue of kurtosis represented by three alternative distributions viz., normal distribution, standardized Student’s *t*–distribution and standardized GED, we first note that that the volatility parameters i.e., α_1 and δ of GARCH(1,1) specification, for example, are significant under each of the three distributions considered. It is also evident that the excess conditional kurtosis, v, which is a parameter for both the *t*–distribution and GED, is significantly present in all the volatility models. For the GARCH(1,1) model, the estimates of v are 7.219 and 1.423 for the *t*–distribution and GED, respectively. The -ratios values are 9.396 and 36.800, respectively. Further, the estimate of $\hat{\alpha}_1$, the parameter representing volatility clustering, is significantly pronounced under all the three distributions. For instance, we find that is 0.158 under normality, whereas for *t*–distribution and GED, the corresponding values are 0.144 and 0.146, respectively.

As regards the suitability of the distributional assumptions under a specific volatility model for returns, we note that the density of standardized GED reduces to that of normal when the shape parameter takes the value 2 i.e., $v = 2$, and hence the latter belongs to the family of GED. Likewise, normal belongs to the family of standardized *t*–distribution since the density of the latter coincides with the former as $v \to \infty$, where v now stands for the degrees of freedom. We can, therefore, carry out formal tests like the likelihood ratio (LR) test to find if normality is the appropriate distributional assumption against the alternative of GED. In a similar spirit, such a test can be envisaged for normality against the alternative of *t*–distribution. However, a formal test is not possible since from the point of view of a proper test of hypothesis, the underlying null hypothesis H_0: $v \to \infty$ is not well specified. Obviously, such a test involving *t*–distribution and GED cannot be performed at all.
Now, the maximized log-likelihood values for the three families of distributions viz., Gaussian, *t* and GED, under GARCH(1,1) volatility model have been obtained as 8490.319, 8558.595 and 8543.195, respectively, for the return series. After applying the LR test under the null hypothesis: H_0: $v = 2$ against the alternative hypothesis H_1: $v \neq 2$ under the GED family, we find that the test statistic value is 105.752. This value is very high compared to the χ^2_1 critical value of 6.63 at 1 per cent level of significance, and hence the null hypothesis is strongly rejected in favour of GED. A similar conclusion regarding a comparison between the Gaussian distribution and the standardized *t* distribution can perhaps be drawn heuristically, but not on the basis of a formal test. Thus, we can conclude that the assumption of normality is not an appropriate distributional assumption against the alternatives of GED and *t* under the GARCH(1,1) volatility model. It is obvious from Tables 3.6 through 3.8 that the same conclusion holds for each of the other three volatility models. Therefore, we can conclude that, as evinced by each of the volatility models considered here, the conditional distributional assumption of normality is not quite statistically tenable for comprehending returns on BSESENSEX.

Finally, we note that, as in the case of GARCH(1,1) specification, estimated values of under all the three distributions indicate that the excess kurtosis of the conditional distribution is significantly present for each model. Computational results also show that the maximum log-likelihood values for the three assumed distributions of normal, *t* and GED for TGARCH(1,1) are 8518.586, 8574.205 and 8559.084, respectively. Applying the LR test, it is evident that as in the case of GARCH(1,1), the assumption of normal distribution is rejected against GED and, heuristically speaking, against the alternative of *t* distribution as well. This is true for the other two volatility models as well.

4. FORECAST PERFORMANCE

In this section, the performances of the models in terms of in-sample and out-of-sample forecasts of returns r_t using the criteria of MAE, MAPE, RMSE and Theil's inequality are discussed[1]. As already mentioned, in-sample data range from January 1996 to December 2008, and the hold-out sample covers the period January 2009 to December 2010. We have applied the dynamic forecasting technique to calculate multi-step forecasts starting from the first period in the hold-out sample.

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[1] The criterion of AMAPE could not be computed since this in not available in EVIEWS 7.

	In-sample				Out-of-sample			
Criterion / Model	MAE	MAPE	RMSE	Theil's coefficient	MAE	MAPE	RMSE	Theil's coefficient
GARCH Normal	0.013	165.247	0.018	0.839*	0.010	150.320	0.015	0.698*
GARCH *t*	0.013	164.885	0.018	0.841*	0.010	150.460	0.016	0.704*
GARCH GED	0.013	166.281	0.018	0.839*	0.010	150.685	0.015	0.699*
TGARCH Normal	0.013	153.285	0.018	0.849	0.010	139.744	0.016	0.712
TGARCH *t*	0.013	156.892	0.018	0.850	0.011	142.813	0.016	0.716
TGARCH GED	0.013	158.477	0.018	0.846	0.010	144.042	0.016	0.710
EGARCH Normal	0.013	147.751	0.018	0.848	0.010	132.113	0.015	0.711
EGARCH *t*	0.013	157.137	0.018	0.847	0.010	142.079	0.016	0.711
EGARCH GED	0.013	158.364	0.018	0.845	0.010	142.595	0.016	0.708
Asymmetric PARCH Normal	0.013	139.423*	0.018	0.854	0.010	125.880*	0.016	0.721
Asymmetric PARCH *t*	0.013	151.943*	0.017	0.855	0.010	133.799*	0.016	0.720
Asymmetric PARCH GED	0.013	154.613*	0.017	0.847	0.010	139.737*	0.016	0.710

Table 3.9 In-sample and out-of-sample forecast performances of returns on BSESENSEX under alternative volatility models and conditional distributions.

Notes: (i) The forecast evaluation criteria used viz., MAE, MAPE and RMSE stand for mean absolute error, mean absolute per cent error, and root mean squared error, respectively.

(ii) * denotes the lowest value among the four models for the same distribution by a particular criterion.

Table 4.9 presents the values of MAE, MAPE, RMSE, and the Theil's inequality for all models arising out of the four volatility models and the three distributions considered. These numerical figures clearly show that MAE and RMSE have produced almost the same value for all of the four volatility models irrespective of the distributional assumption. For instance, in case of in-sample forecasting, the MAE values under the Gaussian distribution is 0.013 for all the GARCH(1,1), TGARCH(1,1), EGARCH(1,1), and asymmetric PARCH(1,1) models. The corresponding RMSE values are 0.018, 0.017, 0.018 and 0.018. The in-sample MAPE value is minimum for the asymmetric PARCH model, whereas the Theil's inequality is minimum for GARCH model. The out-of-sample performance also shows similar findings. It may thus be concluded that for returns based on BSESENSEX, asymmetric PARCH(1,1) performs better than the other models by these forecasting criteria.

Thus, considering both the maximized log-likelihood values and the in-sample and out-of-sample forecasting performances, we can conclude that the asymmetric PARCH and TGARCH perform equally well for Indian stock returns on BSESENSEX.

5. CONCLUSIONS

In this paper, we have empirically examined the twin issues of asymmetry/leverage effect and excess kurtosis, both of which are highly prevalent in stock returns, for returns on BSESENSEX, the most important and the premier stock index of India. To that end, apart from the Gaussian distribution and GARCH volatility specification, we have considered (i) three alternative models of volatility viz., the EGARCH, TGARCH and asymmetric PARCH for representing the phenomenon of 'leverage effect' in returns, and (ii) two alternative conditional distributions for the innovations – standardized Student's *t*–distribution and standardized GED – so that the leptokurtic property of the return distribution are captured. To find which of the four assumed volatility models best describes the volatility prevalent in returns on BSESENSEX, standard forecast evaluation criteria like the MAE, MAPE, RMSE and Theil's inequality have been used, considering both in-sample and out-of-sample forecasts.

In our empirical analysis, we find a significant presence of the 'leverage effect' in the Indian stock returns through each of the three models considered. The values of MAE and RMSE, for in-sample as well as the hold-out sample, clearly demonstrate that, under a Gaussian distribution, all the volatility models considered viz., GARCH(1,1), TGARCH(1,1), EGARCH(1,1) and asymmetric PARCH, perform almost equally well under a consideration of both of these criteria. However, in both in-sample and out-of-sample, MAPE produces the minimum value for the asymmetric PARCH model, whereas the Theil's inequality is minimum for the GARCH model. Thus, it can be concluded that while there is practically no difference in terms of the chosen criteria between the EGARCH and TGARCH specifications, when compared with the other two volatility models, the standard GARCH model is the best in terms of Theil's measure, and the asymmetric PARCH model in terms of MAPE. Regarding 'leverage effect' as an important phenomenon for the Indian stock market, given the poor reliability of Theil's measure, we can conclude that the asymmetric PARCH model is a slightly better representative model compared to the simple GARCH model, EGARCH and TGRACH models.

Our results indicate that the volatility parameters are significant for all the four volatility models under all the three distributions considered. Further, it is also evident that the excess kurtosis, which is a parameter in both *t* distribution and GED, is significantly present in all the four volatility models, thereby indicating that the standard conditional normality assumption is not adequate. We have also found that the assumption of normality for the conditional distribution is not quite tenable statistically against GED and also perhaps against *t*–distribution under all the volatility models considered viz., GARCH(1,1), TGARCH(1,1), EGARCH(1,1) and asymmetric PARCH.

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