Infinite-Variance Error Structure in Finance and Economics

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ABSTRACT

Many macroeconomic and financial data exhibit large outliers and high volatility so that their returns are usually modeled to follow an infinite-variance stable process. Extreme behaviors in such data tend to exist especially for emerging markets due to frequent existence of high economic turmoil. A relatively new area of research studies that model the financial returns as infinite-variance stable errors exists for emerging markets as well as for industrialized countries. This study aims to briefly introduce the reader the concept of infinite-variance stable distributions, discuss some existing studies on unit root and co-integration tests that assume infinite-variance stable error structure, and then to point out the potential lines of research while showing the significance of this relatively new concept.

Key words: *Infinite-Variance Errors, Stable Distributions, Financial Returns, Unit Root Tests, Co-Integration Tests*
JEL Classifications: C21, C22, C32

1. INTRODUCTION

Normality of the error terms in econometric applications is a widely used assumption. Because each error term (usually denoted as u_t or ε_t) in regression and time series models can be hypothesized as the sum of a large number of factors. If those factors are independent with finite-variance and none of them is "too large" compared to others, then each error term is normally distributed by the Central Limit Theorem (CLT).

Recently it has been observed that the behaviors of many random processes that affect some important economic data do not conform well with the characteristics of the normal distribution. The need of heavier-tailed or leptokurtotic distributions emerged to capture the higher variability that exists in the data. Thus research emphasis has shifted towards the stable distributions to model the error terms.

Stable distributions are preferred over many other heavier-tailed distributions than the Gaussian normal, such as the *t*-distribution, because the Generalized Central Limit Theorem (GCLT) applies to them[1]. It has been shown that the stable distributions are the only possible limit law for sums of independent and identically distributed random variables when they are properly normalized and centered. This sounds like the ordinary CLT but it requires a finite second moment whereas GCLT can be generalized to infinite-variance behavior.

2. STABLE DISTRIBUTIONS

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[1] For a detailed analysis on stable distributions the reader is referred to Samorodnitsky and Taqqu (1994).

The reason why these distributions are called stable distributions is because the stable random variables are closed under linear combinations (stability property). Stable or α-stable distributions are characterized by four parameters: $\alpha, \beta, \sigma, \mu$. $\alpha \in (0,2]$ is the stability index or characteristic exponent, and is an important parameter which determines the tail heaviness/thickness. If $\alpha = 2$ it corresponds to the finite-variance case; the distribution is Gaussian normal. As α gets smaller tails get heavier and second moment is infinite, variance ceases to exist. If $\alpha = 1$, the distribution is Cauchy. $\beta \in (-1, 1)$ is the skewness parameter which allows for asymmetry in the distribution. If $\beta = 0$ stable distribution is symmetric. $\sigma > 0$ is the scale parameter and μ is a real number and location parameter.

Intuitively, financial returns having infinite-variance comes from the fact that the financial return data are defined over an infinite range $(-\infty, +\infty)$. Over that range stable distributions assign high probabilities to the tails so that the second moment becomes infinite. In statistical theory, if X is a random variable coming from the stable distribution which is denoted as $X \sim S_\alpha(\beta, \sigma, \mu)$, one can write the second moment of X as $E|X|^2$:

$$E|X|^2 = \int_{-\infty}^{\infty} |x|^2 f(\mathrm{x})dx$$

where $f(.)$ is the pdf of X. As the tails get heavier for $\alpha \leq 1$, even the first moment or mean ceases to exist. Probability density functions of stable variables exist and are continuous over $(-\infty, +\infty)$. However, except for three special cases, their closed forms are not known. Those three cases are Cauchy distribution ($\alpha = 1, \beta = 0$), Gaussian normal distribution ($\alpha = 2, \beta = 0$), and Lévy distribution ($\alpha = 0.5, \beta = \pm 1$).

In general, if the researcher is suspicious of a leptokurtotic behavior in financial return data, there are many ways to estimate the parameter α which indicates whether the tail heaviness is more than the normal distribution's[2]. Maximum likelihood estimation technique gives accurate and reliable results (see for example Nolan, 2001)[3]. Also there is the Hill estimator proposed by Hill (1975). Both methods are widely used for stability index estimation.

3. IMPLICATIONS TO FINANCE AND ECONOMICS

Usage of stable distributions has been appealing to the researchers in many diverse fields including finance, economics, network traffic modeling, signal processing, and oceanography[4].

Stable distributions' importance in the finance and economics fields mostly emerge from the belief that many financial and economic returns are the aggregation or summation of vast number of independent small shocks defined over an infinite range, which, are influenced by the arriving of new information and the decisions of market participants (McCulloch, 1996). By GCLT, we can claim that the only possible limiting distributions of some important economic or financial returns (including stock price changes, inflation rates, interest rate changes and exchange rate changes) have to be the stable ones (Rachev et al., 2007).
In his seminal paper Mandelbrot (1963) studies the cotton prices in the United States and shows that the logarithmic price changes behave more like an infinite-variance stable distribution than

[2] There are numerous ways to identify heavy-tailed behavior in a time series. Particularly if one's aim is to find out the deviation from normality, checking the Q-Q and P-P plots is always a basic and helpful method to analyze data. Also as mentioned in the text, estimating the stable parameters (Nolan, 2001) especially the stability index, will guide the researcher to detect the heavy-tailed phenomenon.
[3] In fact, all the stable parameters can be estimated via the Maximum Likelihood Estimation. Consistency and asymptotic normality of the Maximum Likelihood Estimators are shown in DuMouchel (1973).
[4] See for example Serttaş (2011) and the references therein.

a normal distribution. Fama (1965) studies the stock prices and arrives at a similar conclusion strengthening Mandelbrot's claims. Mandelbrot (1967) shows additional empirical evidence for wheat and railroad stock price changes, as well as for interest rate returns, and exchange rate returns. Provided that there exist large number of extreme values in these returns, a normal approximation would not suffice.

The returns can be modeled as:

$$p_t = p_{t-1} + \varepsilon_t$$

where p_t is the logarithm of the price of the commodity, exchange rate, or interest rate. ε_t or the error term is the price return which is modeled as following a stable distribution.

In literature, Westerfield (1977), Bagshaw and Humpage (1986), So (1987), and Falk and Wang (2003) study the foreign exchange rate changes as stably distributed. The studies that find evidence of infinite-variance in exchange rate returns are Koedijk et al. (1990), Koedijk and Kool (1992), Akgiray et al. (1988), Fofack and Nolan (2001) and Basterfield et al. (2003). McCulloch (1985), Charemza et al. (2005) and Bidarkota and McCulloch (1996) consider interest rate returns and inflation rates within the stable distribution context. Falk and Wang (2003) calculate the stability indices of inflation rates for 12 industrialized countries and find infinite-variance behavior.

The importance of stable distributions for the data of emerging markets is worth a consideration because emerging countries tend to be more subject to frequent and pronounced external and internal shocks than the developed ones (Ibragimov et. al., 2013). Thus their data are prone to higher volatility compared to the industrialized ones. Recent papers by Ibragimov and Khamidov (2010) and Ibragimov et. al. (2013) explore the heavy-tailedness phenomenon specifically concentrating on the emerging markets, while Kabaśinskas et. al. (2009) consider the phenomenon for the Baltic States.

4. ECONOMETRIC STUDIES ON INFINITE-VARIANCE STABLE ERRORS

There are a plethora of theoretical studies that model the error structure as stably distributed. Under regression context, stable error structure has been analyzed for different estimators (see for example Phillips (1995), Thavaneswaran and Peiris (1999), Kurz-Kim and Loretan (2014) and the references therein).

One line of theoretical studies that makes use of the stable errors assumption is on unit-root tests and a related one is on co-integration tests. There are many studies on unit root tests including those of Chan and Tran (1989), Phillips (1990), Rachev et al. (1998), Patterson and Heravi (2003), and Samarakoon and Knight (2009). Co-integration tests for infinite-variance error structure are for example considered in Caner (1998), Paulauskas and Rachev (1998), Chen and Hsiao (2010) and Serttaş (2011).

4.1. Unit Root Tests

Suppose a time series y_t is generated as:

$$p_t = p_{t-1} + \varepsilon_t$$

with $\beta = 1$ (unit root process[5]) for $\varepsilon_t \sim$ i.i.d. $S_\alpha(\beta, \sigma, \mu)$ and $\alpha < 2$. Chan and Tran (1989) analyzes the aymptotics of the least squares estimator which we denote as b_{OLS} and derives the limiting distribution of $T(b_{\mathrm{OLS}} - 1)$ as a function of Lévy process, where T is the sample size. Phillips (1990) provides generalizations to their results for weakly dependent errors, also discussing extensions to models with drifts and time trends. Phillips (1990) is the infinite-variance extension of Phillips and Perron (1988); the proposed unit root tests in that study are referred to as Phillips-Perron type unit root tests. Phillips (1990) also studies the asymptotics of the usual t-statistic[6]. Rachev et al (1998) investigates the asymptotic distributions of b_{OLS} with infinite-variance errors in such a regression:

$$y_t = \mu + \beta y_{t-1} + \varepsilon_t$$

where μ is the drift term. They also derive the asymptotics of the OLS estimator for μ and the usual t-statistics t_β and $t_{\beta,\mu}$. Patterson and Heravi (2003) contributes to the asymptotic analysis of unit root tests under infinite-variance assumption (in particular for Cauchy distribution from the stable family) by considering the augmented Dickey-Fuller (ADF) unit root tests for an AR(p) process in the form[7]:

$$\Delta y_t = \gamma y_{t-1} + \textstyle\sum_{j=2}^{p} \theta_j \Delta y_{t-j+1} + \varepsilon_t$$

If unit root exists then $\gamma = 0$.

These studies so far concentrate on the least squares estimator. Samarakoon and Knight (2009) consider M-estimators in testing unit root processes of AR(p) form which are driven by infinite variance shocks. M-estimators are a broad class of estimators which include the least absolute deviation (LAD) estimator as a special case. LAD estimator is a robust estimator; it is more robust to outliers and heavy-tails than the OLS estimator. The least squares (OLS) estimator is also in the class of M-estimators. In particular, for a simple regression as given below,

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

β_0 and β_1 can be estimated by minimizing the objective function:

$$\textstyle\sum_{t=1}^{n} \rho(y_t - \beta_0 - \beta_1 X_t)$$

For LAD estimator the function $\rho(X) = |X|$ and for OLS estimator $\rho(X) = X^2$. OLS-based infinite-variance error unit root tests are studied in Chan and Tran (1989), Phillips (1990), and Rachev et al. (1998). Those studies derive the asymptotic distributions of the conventional unit root test statistics under the new assumptions and show that the limiting distributions depend on the stability index or the characteristic[8] exponent α. Form those studies, one can deduce that the usage of finite-variance critical values of Dickey-Fuller and Phillips-Perron type unit root tests will lead to size distortions if the data exhibit infinite-variance behavior (Rachev and Mittnik, 2000). Thus the infinite-variance critical values should be used for correctly-sized unit root tests[9].

[5] If $\beta = 1$; series y^t is a unit root process, it is also an integrated process of order one or I(1).

[6] The t-statistics for testing for a unit root in a time series without drift, with drift and time trends have been proposed in Dickey and Fuller (1979), the tests are often referred to as Dickey-Fuller type unit root tests. See the paper for more details

[7] See Said and Dickey (1984) on ADF unit root tests.

[8] Samarakoon and Knight (2009) show a similar result for the unit root tests they propose for the M-estimators under infinite-variance.

[9] See for example Charemza et al. (2005) for an empirical application.

A recent study by Cavaliere et al (2016) on unit root tests with infinite-variance errors focus on the OLS estimator and extend on the works of Chan and Tran (1989) and Samarakoon and Knight (2009) to investigate the asymptotic theory of the ADF tests where the shocks ε_t follow a linear process which itself is driven by infinite-variance stable innovations ξ:

$$\varepsilon_t = \sum_{i=1}^{\infty} \pi_i \, \xi_{t-i}$$

Georgiev et al (2017) analyze the theoretical and practical impact of heavy-tailed innovations on some commonly used unit root tests including ADF and Phillips-Perron tests. They assume a data generating process of a near-integrated time series driven by a linear shock process which is affected by infinite-variance stable errors[10]. Their simulation results suggest only mild distortions exist if one uses the standard critical values of the finite-variance tests. They also show that under their assumptions, a variant of the ADF tests based on the use of Eicker-White standard errors can lead to significant power improvements relative to the conventional unit root tests.

4.2. Co-Integration Tests

Many financial and economic time series are non-stationary processes which have trend and cyclical components. A necessary condition for a set of non-stationary variables to have a long-run equilibrium relationship is that they are co-integrated. A vector of time series $\boldsymbol{v}_t$, which is formed of n individual time series, is co-integrated if those individual series are difference stationary and their linear combination $\boldsymbol{\delta}'\boldsymbol{v}_t$ is stationary[11]. In other words, $\boldsymbol{v}_t$ is co-integrated with the co-integrating vector $\boldsymbol{\delta}$ if a linear combination of the series forming $\boldsymbol{v}_t$ (which is $\boldsymbol{\delta}'\boldsymbol{v}_t$) is integrated with a lower order than the individual series. It is possible to have more than one co-integrating vectors for $n > 2$. In that case, it can be assumed that there are $r \leq n - 1$ linearly independent co-integrating vectors and the number of co-integrating vectors is the co-integrating rank of $\boldsymbol{v}_t$. One can possibly categorize the most popular approaches in testing co-integration into two: residual-based co-integration tests (Engle and Granger, 1987; Phillips and Ouliaris, 1990) and likelihood ratio tests (Johansen, 1988, 1991).

The proposal of the residual-based approach is to estimate the co-integrating relationship and use one of the unit root tests to check for the stationarity of the residuals. The method is implemented by choosing one of the variables of $\boldsymbol{v}_t$ as the dependent variable; say y_t, and the rest as explanatory variables, say vector x_t. Then it involves two steps; deriving the residuals from the regression below,

$$y_t = \delta \, x_t + u_t$$

As a next step, a unit root test is applied on the residuals (generally after running OLS to estimate the equation) to check for the stationarity of the residuals. Based on their power comparisons Engle and Granger (1987) suggest performing an augmented Dickey-Fuller test to test for a unit root in the residuals. Engle and Granger (1987) assume that the error processes driving x_t and y_t series are i.i.d. normal. Phillips and Ouliaris (1990) assume the error terms to be finite-variance and strict stationary processes.

To test for co-integration through likelihood ratio tests, the studies of Johansen (1988, 1991) are generally cited. While testing for co-integration in Johansen's approach, one assumes a parametric model (for example, VAR model) for $\boldsymbol{v}_t$ and tests for co-integration within that model. Johansen's trace and maximum eigenvalue test statistics usually denoted as λ_{trace} and

[10] This framework is in between the pure finite-variance and pure infinite-variance assumption.

[11] A more formal co-integration definition exists in Engle and Granger (1987).

λ_{max}, allow us to test for the existence of co-integration and to test for the number of co-integrating vectors of $\boldsymbol{v}_t$.

Infinite-variance error structure is also explored for co-integration tests in the literature. Caner (1998) develops the asymptotic theory for residual-based co-integration tests due to Phillips and Ouliaris (1990) and for Johansen (1988, 1991) type tests of co-integration under the assumption of infinite-variance errors extending the results of Phillips and Ouliaris (1990) and Johansen (1988, 1991) which consider only finite-variance error structure. It is shown in Caner (1998) that aymptotic distributions of all the test statistics of those tests depend on the stability index α. Thus size distortions arise if one uses the finite-variance critical values of the co-integration tests of Phillips and Ouliaris (1990) and Johansen (1988, 1991) under infinite-variance error structure. Similar to unit root tests, when applying co-integration tests to heavy-tailed data with stable errors $\alpha < 2$, it is suggested that the infinite-variance critical values should be used for size correction.

Some theoretical papers that focus on infinite-variance stable errors to extend the various conventional OLS–based co-integration tests are Caner (1998), Paulauskas and Rachev (1998) and Chen and Hsiao (2010). Caner (1998) considers co-integration tests for symmetric stable innovations with independent components of the same stability index. Paulauskas and Rachev (1998) generalize the asymptotic theory for integrated processes of general stable innovations, even for the case of different stability indices, while discussing residual-based tests. Chen and Hsiao (2010) contribute to the literature by generalizing the Johansen type tests to the case of symmetric stable innovations with discrete spectral measures.

Serttaş (2011) proposes new residual-based co-integration tests while implementing LAD-based estimations for infinite-variance error structure that are in the domain of attraction of a stable law. The study shows that the critical values of the new tests depend on the stability index and finds that the LAD-based tests have power advantages over the standard OLS-based tests as the sample size gets larger and the tails get heavier. Knight and Samarakoon (2009) study the limiting distributions of M-estimators in co-integrated models with infinite-variance errors for Johansen approach and show that the convergence of M-estimators are faster than the OLS.

5. CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

This paper aims to familiarize the reader with the concept of stable distributions, their importance in the finance & economics field and to present the existing literature of unit root and co-integration tests for infinite-variance stable errors[12]. The literature on stable distributions has started with the early works of Mandelbrot (1963) and Fama (1965). Since then, many theoretical and applied econometric papers relied on the assumption of stable error structure and a vast number of studies published in the area.

In literature, many theoretical papers exist that deal with unit root and co-integration tests under infinite-variance error assumption. Section 4.1 discusses the extant unit root tests which are extensions of the standard ADF tests and Phillips-Perron tests. Section 4.2 discusses the existing studies on co-integration tests which are infinite-variance extensions of residual-based and Johansen type tests.

[12] Also under macroeconomics context, Vector Autoregression (VAR) and Structural Vector Autoregression (SVAR) models are analyzed under stable errors where the disturbances in one or more of the equations in the VAR form might have infinite variance for macro data (Hannsgen ,2008; Zarepour and Roknossadati, 2008; Hannsgen, 2011).

Although there are many studies on unit root and co-integration tests assuming infinite-variance errors, the emphasis is mainly on least-squares based estimation methods. Least squares estimator is commonly used in applications because it has a closed form solution and is easy to implement. However, in case of heavy-tailed error structure, OLS estimator tends to focus on a few large errors and ignore the rest of the data (Wilson, 1978; Calder and Davis, 1998). Extensions of OLS-based tests towards using M-estimators (also includes the robust LAD estimator as a special case) has been done by researchers recently in the literature (Samarakoon and Knight, 2009; Knight and Samarakoon, 2009; Serttaş, 2011). Theoretical extensions of existing OLS-based tests (some of which have been mentioned in the text) to M-estimators are potential research areas. For example, Samarakoon and Knight (2009) extend the ADF type unit root tests to M-estimators; one may also consider a similar extension of Phillips-Perron type unit root tests to M-estimators. Also various simulation experiments can be implemented for conventional finite-variance unit root and co-integration tests by replacing the errors with their infinite-variance counterparts to check how the tests perform with the replaced errors.

Furthermore, many of the theoretical papers mentioned until now that make the assumption of stable errors have empirical application potentials in practice as the high frequency finance and economics data for developed and developing/emerging markets tend to exhibit asymmetric and leptokurtotic behavior. Usually in addition to asymmetry and heavy-tailed structure, returns tend to have high peaknesses around their means. This feature is also better captured in the stable distribution context.

As more and more economics and finance data become available each day for the emerging markets and the research options increase for their data, an interesting line of research with stable distributions shifts the consideration of researchers toward the emerging markets. This momentum in the field also deserves attention and is worth exploring.

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